Decentralized adaptive synchronization in nonlinear dynamical networks with nonidentical nodes

Alexander L. Fradkov and Ibragim A. Junussov *†‡

December 11, 2009

Abstract

For a network of interconnected nonlinear dynamical systems an adaptive leader-follower output feedback synchronization problem is considered. The proposed structure of decentralized controller and adaptation algorithm is based on speed-gradient and passivity. Sufficient conditions of synchronization for nonidentical nodes are established. An example of synchronization of the network of nonidentical Chua systems is analyzed. The main contribution of the paper is adaptive controller design and analysis under conditions of incomplete measurements, incomplete control and uncertainty.

1 Introduction

Adaptive synchronization of networked dynamical systems has attracted a growing interest during recent years [1–4]. It is motivated by a broad area of potential applications: formation control, cooperative control, control of power networks, communication networks, production networks, etc. Existing works [1–4] and others are dealing with full state feedback and linear interconnections. The solutions are based on Lyapunov functions formed as sum of Lyapunov functions for local subsystems. As for adaptive control algorithms they are based on either local (decentralized [5–12]) or nearest neighbor (described by an information graph [13–16]) strategies.

Despite a great interest in control of network, only a restricted class of them is currently solved. E.g. in existing papers mainly linear models of subsystems are considered [13, 14]. In nonlinear case only passive or passifiable systems are studied and control is organized according to information graph, i.e. not completely decentralized [15, 16]. Availability of

*A. L. Fradkov is with the Institute for Problems of Mechanical Engineering, Russian Academy of Sciences, 61 Bolshoy ave V.O., St. Petersburg, 199178, Russia, fax: +7(812)321-4771, e-mail: fradkov@mail.ru

†I. A. Junussov is with Department of Theoretical Cybernetics, St.Petersburg State University, Universitetsky prospekt, 28, Peterhof, 198504, St. Petersburg, Russia, e-mail: dxdtfxut@gmail.com

‡The work was supported by the Russian Foundation for Basic Research (RFBR), grants 07-01-92166, 08-01-00775; by Russian Academy of Sciences (RAS) youth research program; by the program of basic research of OEMPPU RAS #2 "Control and safety in energy and technical systems"; by the Council for grants of the RF President to support young Russian researchers and leading scientific schools (project NSh-2387.2008.1).

the whole state vector for measurement as well as appearance of control in all equations for all nodes is assumed in decentralized stability and synchronization problems [1–4]. Powerful passivity based approaches are not developed for adaptive synchronization problems.

In this paper we consider the problem of master-slave (leader-follower) synchronization in a network of nonidentical systems in Lurie form where system models can be split into linear and nonlinear parts. Case of identical nodes is studied in [17]. Linearity of interconnections is not assumed; links between subsystems can also be nonlinear. In the contrary to known works on adaptive synchronization of networks, see [3, 4], only some output function is available and control appears only in a part of the system equations. It is also assumed that some plant parameters are unknown.The leader subsystem is assumed to be isolated and the control objective is to approach the trajectory of the leader subsystem by all other ones under conditions of uncertainty. Interconnection functions are assumed to be Lipschitz continuous.

The results of [11,12] are employed to solve the posed problem. Adaptation algorithm is designed by the speed-gradient method. It is shown that the control goal is achieved under leader passivity condition, if the interconnection strengths satisfy some inequalities.

The results are illustrated by example of synchronization in network of nonidentical Chua curcuits.

2 Auxiliary results

2.1 Yakubovich-Kalman Lemma

We need Yakubovich-Kalman Lemma in following form, see [18].

Lemma 1 *Let A, B, C be $n \times n, n \times m, n \times m$ real matrices and $u \in \mathbb{R}^m$, $\chi(s) = C^{\mathrm{T}}(sI_n - A)^{-1}B, \operatorname{rank} B = m$. Then the following statements are equivalent:*

1) there exists matrix $H = H^{\mathrm{T}} > 0$ such that

$$HA + A^{\mathrm{T}}H < 0, HB = C; \tag{1}$$

2) polinomial $\det(sI_n - A)$ *is Hurwitz and following frequency domain conditions hold*

$$\operatorname{Re} u^{\mathrm{T}}\chi(i\omega)u > 0, \qquad \lim_{\omega\to\infty} \omega^2 \operatorname{Re} u^{\mathrm{T}}\chi(i\omega)u > 0$$

for all $\omega \in \mathbb{R}^1$, $u \in \mathbb{R}^m, u \neq 0$.

2.2 Speed gradient algorithm in decentralized control

In order to present the main syncronization result of this paper we need to formulate problem statement of decentralized control and Theorem 2.18 from [11] which can also be derived from Theorem 7.6 from [12].

Consider[1] a system $\mathcal{S}$ consisting of d interconnected subsystems $\mathcal{S}_i$, dynamics of each being described by the following equation:

$$\dot{x}_i = F_i(x_i, \tau_i, t) + h_i(x, \tau, t), \qquad i = 1, \dots, d, \tag{2}$$

[1] In this paper norms are Euclidean, $\operatorname{col}(x_1, \dots, x_d)$ stands for column vector with components consisting of components of $x_i, i = 1, \dots, d$.

where $x_i \in \mathbb{R}^{n_i}$ – state vector, $\tau_i \in \mathbb{R}^{m_i}$ - vector of inputs (tunable parameters) of subsystem, $x = \mathrm{col}(x_1, \dots, x_d) \in \mathbb{R}^n$, $\tau = \mathrm{col}(\tau_1, \dots, \tau_d) \in \mathbb{R}^m$ - aggregate state and input vectors of system S, $n = \sum n_i$, $m = \sum m_i$. Vector-function $F_i(\cdot)$ describes local dynamics of subsystem S_i, and vectors $h_i(\cdot)$ describe interconnection between subsystems.

Let $Q_i(x_i, t) \geq 0, i = 1, \dots, d$ be local goal functions and let the control goal be:

$$\lim_{t\to\infty} Q_i(x_i, t) = 0, \qquad i = 1, \dots, d. \tag{3}$$

For all $i = 1, \dots, d$ we assume existence of smooth vector functions $x_i^*(t)$ such that $Q_i(x_i^*(t), t) \equiv 0$, i.e. $x_i^* = \operatorname{argmin}_{x_i} Q_i(x_i, t)$. Decentralized speed-gradient algorithm is introduced as follows:

$$\dot{\tau}_i = -\Gamma_i \nabla_{\tau_i} \omega_i(x_i, \tau_i, t), \qquad i = 1, \dots, d, \tag{4}$$

where

$$\omega_i(x_i, \tau_i, t) = \frac{\partial Q_i}{\partial t} + \nabla_{x_i} Q_i(x_i, t)^{\mathrm{T}} F_i(x_i, \tau_i, t),$$

$\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$, $m_i \times m_i$ - matrix.

Theorem 1 Suppose the following assumptions hold for the system $\mathcal{S}$:

1. Functions $F_i(\cdot)$ are continuous in x_i, t, continuously differentiable in τ_i and locally bounded in $t > 0$; functions $Q_i(x_i, t)$ are uniformly continuous in second argument for all x_i in bounded set, functions $\omega_i(x_i, \tau_i, t)$ are convex in τ_i; there exist constant vectors $\tau_i^* \in \mathbb{R}^{m_i}$ and scalar monotonically increasing functions $\kappa_i(Q_i), \rho_i(Q_i)$ such that $\kappa_i(0) = \rho_i(0) = 0, \lim_{Q_i \to +\infty} \kappa_i(Q_i) = +\infty$
$$\omega_i(x_i, \tau_i^*, t) \leq -\rho_i(Q_i(x_i, t)), \tag{5}$$
and $Q_i(x_i, t) \geq \kappa_i(\|x_i - x_i^*(t)\|)$.

2. functions $h_i(x, \tau, t)$ are continuous and satisfy the following inequalities
$$|\nabla_{x_i} Q_i(x_i, t)^{\mathrm{T}} h_i(x, \tau, t)| \leq \sum_{j=1}^{d} \mu_{ij} \rho_j(Q_j(x_j, t)), \tag{6}$$
where matrix $M - I$ is Hurwitz, $M = \{\mu_{ij}\}$, $\mu_{ij} > 0$, I is identity matrix.

Then system (2),(4) is globally asymptotically stable in variables $x_i - x_i^*(t)$, all trajectories are bounded on $t \in [0, +\infty)$ and satisfy (3).

3 Main result

3.1 Problem statement. Adaptive controller structure

Let the leader subsystem be described by the equation

$$\dot{\overline{x}} = A_L \overline{x} + B_L(\overline{u} + \psi_0(\overline{y})), \quad \overline{y} = C^{\mathrm{T}} \overline{x}, \tag{7}$$

where $\overline{x} \in \mathbb{R}^n$ – state, $\overline{y} \in \mathbb{R}^l$ – measurement, $\overline{u}(t) \in \mathbb{R}^1$ is control that specified in advance, $\psi_0 \colon \mathbb{R}^l \to \mathbb{R}^1$ – internal nonlinearity. Let A_L, B_L, C and $\psi_0(\cdot)$ be known and not depending on the vector of unknown parameters $\xi \in \Xi$, where Ξ is known set.

Consider a network S of d interconnected subsystems S_i, $i = 1, \ldots, d, d \in \mathbb{N}$. Let subsystem S_i be described by the following equation

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + B_L \psi_0(y_i) + \sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(x_i - x_j), \\ y_i &= C^{\mathrm{T}} x_i, \qquad i = 1, \ldots, d, \end{aligned} \tag{8}$$

where $x_i \in \mathbb{R}^n, u_i \in \mathbb{R}^1, \alpha_{ij} \in \mathbb{R}^1, y_i \in \mathbb{R}^l$. Functions $\varphi_{ij}(\cdot)$, $i = 1, \ldots, d$, $j = 1, \ldots, d$, describe interconnections between subsystems. We assume $\varphi_{ii} = (0, 0, \ldots, 0)^{\mathrm{T}}, i = 1, \ldots, d$. Let matrices A_i, B_i and functions $\varphi_{ij}(\cdot)$, $i = 1, \ldots, d, j = 1, \ldots, d$, depend on the vector of unknown parameters $\xi \in \Xi$.

Network model (8) can describe, for example, interconnected electrical generators [19].

Let the control goal be specified as convergence of all subsystems and the leader trajectories:

$$\lim_{t \to +\infty} (x_i(t) - \overline{x}(t)) = 0, \qquad i = 1, \ldots, d. \tag{9}$$

The adaptive synchronization problem is to find a decentralized controller $u_i = \mathcal{U}_i(y_i, \overline{u}, t)$ ensuring the goal (9) for all values of unknown plant parameters.

Denote $\sigma_i(t) = \operatorname{col}(y_i(t), \overline{u}(t))$. Let the main loop of the adaptive system be specified as set of linear tunable local control laws:

$$u_i(t) = \tau_i(t)^{\mathrm{T}} \sigma_i(t), \quad i = 1, \ldots, d, \tag{10}$$

where $\tau_i(t) \in \mathbb{R}^{l+1}, i = 1, \ldots, d$ are tunable parameters. By applying speed-gradient method [12] it is easy to derive the following adaptation law:

$$\dot{\tau}_i = -g^{\mathrm{T}}(y_i - \overline{y}) \Gamma_i \sigma_i(t), i = 1, \ldots, d, \tag{11}$$

where $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$ – $(l+1) \times (l+1)$ matrices, $g \in \mathbb{R}^l$.

3.2 Synchronization conditions

Introduce the following definition.

Definition 1 Let $G \in \mathbb{R}^l$. Function $f \colon \mathbb{R}^l \to \mathbb{R}^1$ is called G-monotonically decreasing if inequality $(x - y)^{\mathrm{T}} G \left(f(x) - f(y)\right) \leq 0$ holds for all $x, y \in \mathbb{R}^l$.

Remark 1. Apparently, for $l = 1, G = 1$ G-monotonical decrease of the function f is equivalent to incremental passivity [21] of the static system with characteristics $(-f)$. Definition 1 is easily extended to dynamical systems with the state vector $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$ and output $y \in \mathbb{R}^l$. It corresponds to existence of a smooth function $V(x_1, x_2)$ satisfying an integral inequality

$$\dot{V}(x_1, x_2) \leq (u_2 - u_1)^{\mathrm{T}} G (y_2 - y_1).$$

The corresponding property can be called incremental G-passivity by analogy with [20].

Consider real matrices $H = H^{\mathrm{T}} > 0, g$ of size $n \times n, l \times 1$ correspondingly and a number $\rho > 0$ such that:

$$HA_L + A_L^{\mathrm{T}} H < -\rho H, \quad HB_L = Cg. \tag{12}$$

Denote $\lambda_* = \lambda_{max}(H)/\lambda_{min}(H)$ condition number of matrix H, where $\lambda_{max}(H), \lambda_{min}(H)$ are maximum and minimum eigenvalues of matrix H.

For analysis of the system dynamics the following assumptions are made.

A1) The functions $\varphi_{ij}(\cdot)$,$i = 1, \dots, d$, $j = 1, \dots, d$ are globally Lipschitz:

$$\|\varphi_{ij}(x) - \varphi_{ij}(x')\| \leq L_{ij}\|x - x'\|, \quad L_{ij} > 0.$$

The function $\psi_0(\cdot)$ is such that the unique existence of solutions of (7) holds.

A2)(Matching conditions, [22]) For each $\xi \in \Xi$ there exist vectors $\nu_i = \nu_i(\xi) \in \mathbb{R}^l$ and numbers $\theta_i = \theta_i(\xi) > 0$ such that for $i = 1, \dots, d$

$$A_L = A_i + B_i \nu_i^{\mathrm{T}} C^{\mathrm{T}}, B_L = \theta_i B_i. \tag{13}$$

Denote $\chi(s) = C^{\mathrm{T}}(sI_n - A_L)^{-1} B_L$. For the case when matrix A_L is Hurwitz introduce notation ρ_* for stability degree of the function's $g^{\mathrm{T}}\chi(s)$ denominator, i.e. $\rho_* = \min_{k=1,\dots,n} |\operatorname{Re} \lambda_k(A_L)|$ where $\lambda_k(A_L)$ are eigenvalues of A_L.

Theorem 2 Let $B_L \neq 0$, matrix A_L be Hurwitz and for some $g \in \mathbb{R}^l$ the following frequency domain conditions hold:

$$\operatorname{Re} g^{\mathrm{T}}\chi(i\omega) > 0, \quad \lim_{\omega \to \infty} \omega^2 \operatorname{Re} g^{\mathrm{T}}\chi(i\omega) > 0 \tag{14}$$

for all $\omega \in \mathbb{R}^1$. Then there exist $H = H^{\mathrm{T}} > 0, \rho > 0$ such that relations (12) hold.

Let for all $\xi \in \Xi$ Assumptions A1, A2 hold, function $\psi_0(\cdot)$ be g-monotonically decreasing, and following inequalities hold

$$\sum_{j=1}^{d} |\alpha_{ij} L_{ij}| < \gamma \quad i = 1, \dots, d, \tag{15}$$

where $\gamma = \rho_*/(4d\lambda_*)$, λ_* is condition number of matrix H.

Then for all $\xi \in \Xi, i = 1, \dots, d$ adaptive controller (10),(11) ensures achievement of the goal (9) and boundedness of functions $\theta_i(t)$ on $[0, \infty)$ for all solutions of the closed-loop system (7), (8), (10), (11).

Proof. Let's apply Lemma 1. Note that in our case $m = 1$, i. e. u is scalar. Let's choose Cg instead of C in (1). Then statement of the Lemma 1 and conditions of Theorem 2 ensure existence of matrix $H = H^{\mathrm{T}} > 0$ such that

$$HA_L + A_L^{\mathrm{T}} H < 0, \quad HB_L = Cg.$$

Now we can conclude that there exists number $\rho > 0$ such that the following is true:

$$HA_L + A_L^{\mathrm{T}} H < -\rho H, \quad HB_L = Cg. \tag{16}$$

Denoting $z_i = x_i - \overline{x}$ introduce auxiliary error subsystems:

$$\begin{aligned} \dot{z}_i =& A_i x_i + B_i u_i + B_L \psi_0(y_i) + \sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(x_i - x_j) - \left(A_L \overline{x} + B_L(\overline{u} + \psi_0(\overline{y}))\right), \\ \tilde{y}_i =& C^{\mathrm{T}} z_i, \qquad i = 1, \dots, d, \end{aligned} \tag{17}$$

here we choose $u_i(t)$ same as in (10).

Let us choose following goal functions $Q_i(z_i) = \frac{1}{2} z_i^{\mathrm{T}} H z_i$, and apply Theorem 1. We need to evaluate the derivative trajectories of $Q_i(z_i)$ along trajectories of isolated (i.e. without interconnections) auxiliary subsystems (17):

$$\omega_i(x_i, \overline{x}, \tau_i) = z_i^{\mathrm{T}} H[A_i x_i + B_i \tau_i^{\mathrm{T}}(t)\sigma_i(t) + B_L \psi_0(y_i) - A_L \overline{x} - B_L(\overline{u} + \psi_0(\overline{y}))]. \tag{18}$$

Denote $\tau_i^* = \mathrm{col}(\nu_i, \theta_i), i = 1, \ldots, d$. By taking $\tau_i = \tau_i^*, i = 1, \ldots, d$, we obtain

$$\begin{aligned}\omega_i(x_i, \overline{x}, \tau_i^*) =& z_i^{\mathrm{T}} H[A_i x_i + B_i(\nu_i C^{\mathrm{T}} x_i + \theta_i \overline{u}) + B_L \psi_0(y_i) - A_L \overline{x} - B_L \overline{u} - B_L \psi_0(\overline{y})] = \\ & z_i^{\mathrm{T}} H[A_L x_i + B_L \overline{u} + B_L(\psi_0(y_i) - \psi_0(\overline{y})) - A_L x_i - B_L \overline{u}] = \\ & z_i^{\mathrm{T}} H[A_L z_i + B_L(\psi_0(y_i) - \psi_0(\overline{y}))].\end{aligned}$$

Further, for $i = 1, \ldots, d$

$$z_i^{\mathrm{T}} H B_L(\psi_0(y_i) - \psi_0(\overline{y})) = z_i^{\mathrm{T}} C g(\psi_0(y_i) - \psi_0(\overline{y})) = (y_i - \overline{y})^{\mathrm{T}} g(\psi_0(y_i) - \psi_0(\overline{y})) \leq 0.$$

The last inequality holds because $\psi_0(\cdot)$ is g-monotonically decreasing. So

$$\omega_i(x_i, \overline{x}, \tau_i^*) \leq \frac{1}{2} z_i^{\mathrm{T}}(H A_L + A_L^{\mathrm{T}} H) z_i.$$

Taking into account (16) we conclude

$$\omega_i(x_i, \overline{x}, \tau_i^*) \leq -\rho\, Q_i(z_i).$$

By taking $\rho_i(Q) = \rho \cdot Q$ we ensure that (5) holds for $i = 1, \ldots, d$. Other conditions from the first part of Theorem 1 hold, since the right hand side of the system (17) and function $Q_i(z_i)$ are continuous in z_i functions not depending in t for any $i = 1, \ldots, d$. Convexity condition is valid since the right hand side of (18) is linear in τ_i.

The interconnection condition (6) in our case reads:

$$|\nabla_{z_i} Q(z_i)^{\mathrm{T}} \sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(z_i - z_j)| \leq \sum_{j=1}^{d} \mu_{ij} \rho \cdot Q(z_j), \tag{19}$$

where $i = 1, \ldots, d$, and matrix $M - I$ should be Hurwitz ($M = \{\mu_{ij}\}$, $\mu_{ij} > 0$).

For the case $d = 1$ we can take $\mu_{11} = 0.5$ and last inequality will be satisfied. Let's consider case $d > 1$.

For $i = 1, \ldots, d$ rewrite (19) as follows:

$$|z_i^{\mathrm{T}} H \sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(z_i - z_j)| \leq \frac{\rho}{2} \sum_{j=1}^{d} \mu_{ij}\, z_j^{\mathrm{T}} H z_j. \tag{20}$$

Evaluate the left-hand side of (20):

$$\begin{aligned}&\left| z_i^{\mathrm{T}} H \sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(z_i - z_j) \right| \leq \sum_{j=1}^{d} |z_i^{\mathrm{T}} H \alpha_{ij} \varphi_{ij}(z_i - z_j)| \leq \\ &\sum_{j=1}^{d} |\alpha_{ij} L_{ij}| \cdot \|z_i\| \cdot \|H\| \cdot \|z_i - z_j\| \leq \sum_{j=1}^{d} |\alpha_{ij} L_{ij}| \cdot \lambda_{max}(H) \cdot (\|z_i\|^2 + \|z_i\| \cdot \|z_j\|),\end{aligned}$$

for $i = 1, \dots, d$. Then for $i = 1, \dots, d$ evaluate lower bound of the right-hand side of (20):

$$\frac{\rho}{2}\sum_{j=1}^{d}\mu_{ij} z_j^{\mathrm{T}} H z_j \geq \frac{\rho}{2}\sum_{j=1}^{d}\mu_{ij}\lambda_{min}(H)\|z_j\|^2.$$

It is seen that for $i = 1, \dots, d$ to ensure (6) it is sufficient to impose an inequality

$$\sum_{j=1}^{d}|\alpha_{ij}L_{ij}|\cdot\lambda_{max}(H)\cdot(\|z_i\|^2+\|z_i\|\cdot\|z_j\|) \leq \frac{\rho}{2}\sum_{j=1}^{d}\mu_{ij}\lambda_{min}(H)\|z_j\|^2,$$

or

$$\sum_{j=1}^{d}|\alpha_{ij}L_{ij}|\cdot(\|z_i\|^2+\|z_i\|\cdot\|z_j\|) \leq \frac{\rho}{2}\frac{\lambda_{min}(H)}{\lambda_{max}(H)}\sum_{j=1}^{d}\mu_{ij}\|z_j\|^2, \qquad i = 1, \dots, d. \tag{21}$$

Denote $\zeta = \rho/(\alpha_{max}\cdot 2\lambda_*)$, where

$$\alpha_{max} = \max_{i:1\leq i\leq d}\sum_{j=1}^{d}|\alpha_{ij}L_{ij}|.$$

Noting that ρ in (12) can be chosen arbitrarily close to ρ_* and taking into account (15) we can conclude that

$$\zeta > 2d.$$

The left-hand side of (21):

$$\sum_{j=1}^{d}|\alpha_{ij}L_{ij}|(\|z_i\|^2+\|z_i\|\cdot\|z_j\|) \leq \left(\sum_{j=1}^{d}|\alpha_{ij}L_{ij}|\right)\cdot\left(\sum_{j=1}^{d}(\|z_i\|^2+\|z_i\|\cdot\|z_j\|)\right) \leq$$
$$\frac{1}{2}\alpha_{max}\left(3d\,\|z_i\|^2+\sum_{j=1}^{d}\|z_j\|^2\right), \quad i = 1, \dots, d.$$

Thus, if following inequality holds then (6) is ensured:

$$3d\,\|z_i\|^2+\sum_{j=1}^{d}\|z_j\|^2 \leq 2\zeta\cdot\sum_{j=1}^{d}\mu_{ij}\|z_j\|^2, \quad i = 1, \dots, d. \tag{22}$$

Introduce matrix $M = \{\mu_{ij}\}$ as follows

$$M = \begin{pmatrix}\mu_{11} & \mu_{12} & \dots & \mu_{1d}\\ \mu_{21} & \mu_{22} & \dots & \mu_{2d}\\ \vdots & \vdots & \ddots & \vdots\\ \mu_{d1} & \mu_{d2} & \dots & \mu_{dd}\end{pmatrix}, \qquad \mu_{ij} = \begin{cases}\frac{1}{2\zeta}(3d+1), & i = j;\\ \frac{1}{2\zeta}, & i \neq j.\end{cases}$$

Such choice of M ensures (22).

Note that M is symmetric. If matrix $I - M$ is positive definite then $M - I$ is Hurwitz. Diagonal elements of $I - M$ are positive since $d > 1$ and $\zeta > 2d$. By taking into account that

$$1 - \frac{1}{2\zeta}(3d+1) - (d-1)\frac{1}{2\zeta} > 0$$

and applying Gershgorin circle Theorem we conclude that $M - I$ is positive definite.

Thus, statement of the Theorem 2 follows from Theorem 1. □

Remark 2. The value of γ can be evaluated by solving LMI (16) by means of one of existing software package.

Remark 3. By interconnections graph of network S we can consider directed graph which is a pair of two sets: a set of nodes and a set of arcs. Cardinality of a set of nodes is d; i-th node is associated with subsystem S_i for any $i = 1, \dots, d$. We say that arc from i-th node to j-th node belongs to the set of arcs if $\varphi_{ij}(\cdot)$ is not zero function. By weighted in-degree of i-th node we define following number: $\sum_{j=1}^{d} |\alpha_{ij} L_{ij}|$. If each nonzero addend from last sum is equal to 1 then introduced definition of weighted in-degree of the node coincides with the definition of in-degree of digraph's node. Thus the inequality (15) can be interpreted as follows: weighted in-degree of each node of interconnections graph must be less than γ.

4 Example. Network of Chua circuits

4.1 System description and theoretical study

Chua circuit is a well known example of simple nonlinear system possessing complex chaotic behavior [23]. Its trajectories are unstable and it is represented in the Lurie form. Let us apply our results to synchronization with leader subsystem in the network of five interconnected nonidentical Chua systems.

Let $m_0 = -8/7, m_1 = -5/7, p = 15.6, q = 30, b = 1$ and $g = 1$.

Let the leader subsystem be described by the equation

$$\dot{\overline{x}} = A_L \overline{x} + B_L(\overline{u} + \psi_0(\overline{y})), \quad \overline{y} = C^{\mathrm{T}} \overline{x},$$

where $\overline{x} \in \mathbb{R}^3$ is state vector of the system, $\overline{y} \in \mathbb{R}^1$ is output available for measurement, $\overline{u}$ is scalar control variable, $\psi_0(\overline{y}) = pv(\overline{y})/b$, where $v(x) = -0.5(m_0 - m_1)(|x+1| - |x-1| - 2x)$. Further, let

$$A_L = \begin{pmatrix} -1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{pmatrix},$$

$B_L = \mathrm{col}(b, 0, 0), C = \mathrm{col}(1, 0, 0)$.

Transfer function $\chi(s) = C^{\mathrm{T}}(sI - A_L)^{-1} B_L = (s^2 + s + 30)/(s^3 + 2s^2 + 31s + 30)$. It is seen from the Nyquist plot of $\chi(i\omega), \forall \omega \in \mathbb{R}^1$, presented on Fig. 1, that first frequency domain inequality of (14) holds. The second frequency domain inequality of (14) also holds since relative degree of $\chi(s)$ is equal to one and highest coefficient of its numerator is positive.

Obviously $\psi_0(\cdot)$ is g-monotonically decreasing.

Let subsystem S_i for $i = 1, \dots, 5$ be described by (8) with $u_i, \alpha_{ij} \in \mathbb{R}^1$. By choosing $(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5) = (3, 1, 4, 1, 5), \theta_i = 1/i, i = 1, \dots, 5$ and using (13) we obtain matrices A_i, B_i for $i = 1, \dots, 5$, which are not equal, i.e. nodes are nonidentical. Denote $\varphi_{ij} = \varphi_{ij}(x_i - x_j), i = 1, \dots, 5, j = 1, \dots, 5$. Let $\varphi_{14}, \varphi_{25}, \varphi_{32}, \varphi_{42}, \varphi_{45}, \varphi_{52}, \varphi_{53}$, be equal

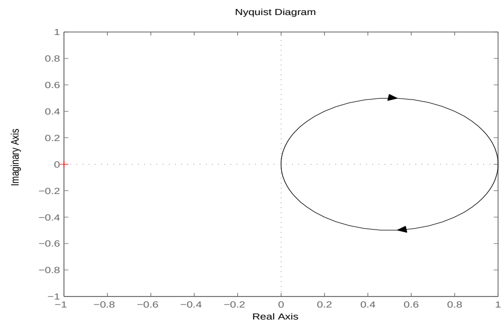

Figure 1: Nyquist plot of $\chi(i\omega), \omega \in \mathbb{R}^1$.

to $(0,0,0)^{\mathrm{T}}$. Further, let

$$\begin{aligned}
\varphi_{12} &= (\sin(x_{11}-x_{21}),0,0)^{\mathrm{T}}, & \varphi_{13} &= (0,x_{12}-x_{32},0)^{\mathrm{T}},\\
\varphi_{15} &= (0,0,\sin(x_{13}-x_{53}))^{\mathrm{T}}, & \varphi_{21} &= (x_{21}-x_{11},0,x_{23}-x_{13})^{\mathrm{T}},\\
\varphi_{23} &= (0,\sin(x_{22}-x_{32}),0)^{\mathrm{T}}, & \varphi_{24} &= (0,x_{22}-x_{42},0)^{\mathrm{T}},\\
\varphi_{31} &= (\sin(x_{31}-x_{11}),0,0)^{\mathrm{T}}, & \varphi_{34} &= (\sin(x_{31}-x_{41}),0,0)^{\mathrm{T}},\\
\varphi_{35} &= (x_{31}-x_{51},x_{32}-x_{52},x_{33}-x_{53})^{\mathrm{T}}, & \varphi_{41} &= (0,\sin(x_{42}-x_{12}),0)^{\mathrm{T}},\\
\varphi_{43} &= (\sin(x_{41}-x_{31}),0,0)^{\mathrm{T}}, & \varphi_{51} &= (x_{51}-x_{11},0,x_{53}-x_{13})^{\mathrm{T}},\\
\varphi_{54} &= (0,x_{52}-x_{42},0)^{\mathrm{T}}.
\end{aligned}$$

Lipschitz constants of all φ_{ij} are equal to 1.

It follows from Theorem 2 that decentralized adaptive control (10) provides synchronization goal (9) if for all $i=1,\dots,5$ inequality $\sum_{j=1}^{5}|\alpha_{ij}| < \gamma$ holds, i.e. if interconnections are sufficiently weak.

4.2 Simulation results

Consider following control of leader subsystem $\overline{u} = \frac{1}{b}\left[(-(1+m_0)p+1)\overline{x}_1 + p\overline{x}_2\right]$. Such $\overline{u}$ ensures chaotic behavior of leader subsystem. Let us put $\Gamma_i = I, i=1,\dots,d$, where I – identity matrix, and

$$\begin{aligned}
&\overline{x}_1(0)=0.5, \quad \overline{x}_2(0)=0, \quad \overline{x}_3(0)=0,\\
&x_1(0)=(7,14,0.4)^{\mathrm{T}}, \quad x_2(0)=(0,4,4)^{\mathrm{T}}\\
&x_3(0)=(1,-1,4.5)^{\mathrm{T}}, \quad x_4(0)=(3,-4,0.2)^{\mathrm{T}}\\
&x_5(0)=(2,8,15).
\end{aligned}$$

Denote by α 5×5 matrix with element α_{ij} lying in the i-th row and the j-th column, $i,j=1,\dots,5$, and

$$\widehat{\alpha} = \begin{pmatrix}
0 & 0.0051 & 0.1395 & 0 & 0.1676\\
0.0662 & 0 & 0.0921 & 0.0065 & 0\\
0.2013 & 0 & 0 & 0.2271 & 0.1430\\
0.0907 & 0 & 0.0675 & 0 & 0\\
0.0663 & 0 & 0 & 0.2773 & 0
\end{pmatrix}.$$

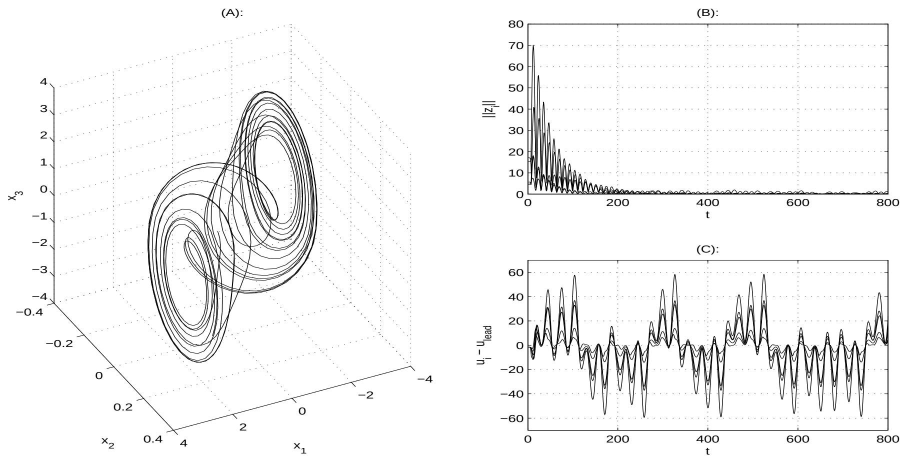

Figure 2: (A): Phase portrait of leader subsystem, (B): $\|z_i\|$, (C): $\tilde{u}_i = u_i - \overline{u}, i = 1, \ldots, 5$.

Let us choose adaptive control $u_i, i = 1, \ldots, 5$ as in (10) and apply Theorem 2. If we take $\alpha = \widehat{\alpha}$, then simulation shows that $\|z_i\| \to 0, i = 1, \ldots, 5$, i.e. synchronization is achieved: all state vectors of nonidentical nodes converge to the state vector of the leader subsystem, see Fig. 2-(B). Phase portrait of the leader subsystem, $\|z_i\|, \tilde{u} = u_i - \overline{u}, i = 1, \ldots, 5$ found by 40 sec. simulation are shown on Fig. 2.

5 Conclusions

In contrast to a large number of previous results, we obtained synchronization conditions for networks consisting of nonidentical nonlinear systems with incomplete measurement, incomplete control, incomplete information about system parameters and coupling. The design of the control algorithm providing synchronization property is based on speed-gradient method [12], while derivation of synchronizability conditions is based on Yakubovich-Kalman lemma and result presented in [11].

References

[1] J. Lu, G. Chen, "A Time-Varying Complex Dynamical Network Model and Its Controlled Synchronization Criteria", *IEEE Trans. Autom.Control*, Vol. 50(6), pp. 841-846, 2005.

[2] J. Yao, D. J. Hill, Z.-H. Guan, H. O. Wang, "Synchronization of Complex Dynamical Networks with Switching Topology via Adaptive Control", in *Proc. 45th IEEE Conf. Dec. Control*, pp. 2819-2824, 2006.

[3] J. Zhou, J. Lu, J. Lu, "Adaptive synchronization of an uncertain complex dynamical network", *IEEE Trans. Autom. Control*, Vol.51(4), pp. 652-656, 2006.

[4] W. S. Zhong, G. M. Dimirovski, J. Zhao, "Decentralized synchronization of an uncertain complex dynamical network", in *Proc. 2007 Amer. Contr. Conf.*, pp. 1437-1442, 2007.

[5] P. A. Ioannou, "Decentralized adaptive control of interconnected systems", *IEEE Trans. on Autom. Control*, Vol. 31(4), pp. 310-314, 1986.

[6] B. M. Mirkin, "Adaptive Decentralized Control with Model Coordination", *Automation and Remote Control*, Vol. 60(1), pp. 73-81, 1999.

[7] D. T. Gavel, D. D. Siljak, "Decentralized adaptive control: structural conditions for stability", *IEEE Trans. on Autom. Control*, Vol. 34(4), pp. 413-426, 1989.

[8] C. Wen, Y. C. Soh, "Decentralized model reference adaptive control without restriction on subsystem relative degrees", *IEEE Trans. on Autom. Control*, Vol. 44(7), pp. 1464-1469, 1999.

[9] S. Jain, F. Khorrami, "Decentralized adaptive control of a class of large-scale nonlinear systems", *IEEE Trans. on Autom. Control*, Vol. 42(2), pp. 136-154, 1997.

[10] Zh. P. Jiang, "Decentralized and adaptive nonlinear tracking of large-scale systems via output feedback", *IEEE Trans. on Autom. Control*, Vol. 45(11), pp. 2122-2128, 2000.

[11] A. L. Fradkov, *Adaptive Control in Complex Systems*, Moscow: Nauka, 1990 (In Russian).

[12] A. L. Fradkov, I. V. Miroshnik, V. O. Nikiforov, *Nonlinear and adaptive control of complex systems*, Kluwer Academic Publishers, Dordrecht, 1999.

[13] J. R. Fax, R. M. Murray, "Information Flow and Cooperative Control of Vehicle Formations", *IEEE Trans. on Autom. Control*, Vol. 49(9), pp. 1465-1476, 2004.

[14] C. Yoshioka, T. Namerikawa, "Observer-based consensus control strategy for multi-agent system with communication time delay", in *Proc. 17th IEEE Intern. Conf. on Control Applications*, pp. 1037-1042, 2008.

[15] N. Chopra, M. W. Spong, "Output Synchronization of Nonlinear Systems with Time Delay in Communication", in *Proc. 45th IEEE Conf. Dec. Control*, pp. 4986-4992, 2006.

[16] N. Chopra, M. W. Spong, "Output Synchronization of Nonlinear Systems with Relative Degree One" in *Recent Advances in Learning and Control*, Vol. 371, Springer-Verlag, pp. 51-64, 2008.

[17] I. A. Dzhunusov and A. L. Fradkov, "Adaptive synchronization of a network of interconnected nonlinear Lur'e systems", *Automation and Remote Control*, Vol. 70(7), pp. 1190-1205, 2009.

[18] V. A. Yakubovich, G. A. Leonov, A. Kh. Gelig *Stability of Stationary Sets in Control Systems with Discontinuous Nonlinearities*, Singapore: World Scientific, 2004.

[19] J. L. Willems, "A partial stability approach to the problem of transient power system stability", *Int. J. of Control*, Vol. 19(1), pp. 1-14, 1974.

[20] A. L. Fradkov, "Passification of nonsquare linear systems and feedback Yakubovich-Kalman-Popov Lemma", Europ. J. of Contr., Vol. 9(6), pp. 577-586, 2003.

[21] A. Pavlov, L. Marconi, "Incremental passivity and output regulation", Systems and Control Letters, Vol. 57(5), pp. 400-409, 2008.

[22] E. Skafidas, A. L. Fradkov, R. J. Evans, I. M. Mareels, "Trajectory approximation based adaptive control for nonlinear systems under matching conditions", *Automatica*, Vol. 34(3), pp. 287-299, 1998.

[23] C. W. Wu, L. O. Chua, "Synchronization in an array of linearly coupled dynamical systems", *IEEE Trans. Circuits and Systems-I.* Vol. 42(8), pp. 430-447, 1995.